MATH 5A - SAMPLE FINAL EXAM

(1) Find the following limits if they exist. If not, why not?

$$(a)\ \lim_{x\to 0^-} \frac{|x|}{x}\cos x \qquad (b)\ \lim_{x\to 2^-} \frac{1-x}{x-2} \qquad (c)\ \lim_{x\to\infty} \frac{x-3}{x^2}$$

(2) Use the difference quotient and definition of derivative to find f'(x) if f(x) = x^3 - x.

(3) Find the derivative of each of the following functions and simplify your answer:

(a) f(x) = $\sqrt{x}\,(x^2+2)$ (b) h(x) = $(1 + \tan^2 x)^3$ (c) g(x) = $\dfrac{x}{\sqrt{x^2+1}}$

(4) Find the y-intercept of the line tangent to the curve $x^2 - xy - y^2 = 1$ at (2,1)

(5) Integrate:

(a) $\displaystyle\int_0^{\pi/4} \sin x \cos^3 x \; dx$

(b) $\displaystyle\int_0^{2} (3-x)^2 \; dx$

(c) $\displaystyle\int \frac{x^3}{\sqrt{x^2-1}} dx$

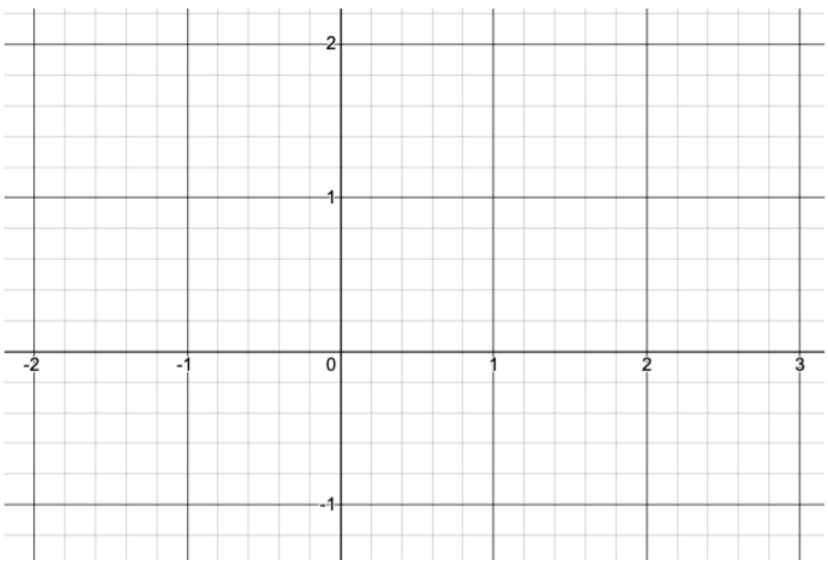
(6) Given f(x) = $x(1-x)^{2/5}$,

(a) find the interval(s) on which the function f is

(i) increasing (ii) decreasing (iii) concave up (iv) concave down

(b) find all critical points (c) inflection points (d) find all extrema

(e) given the above information, sketch a graph of the above function.



(7) A person in a rowboat 2 miles from the nearest point on a straight shoreline wishes to reach a house 6 miles farther down the shore. If the person can row at a rate of 3 mi/hr and walk at a rate of 5mi/hr. find the least amount of time required to reach the house. (Show all steps you used to determine minimum is absolute)

(8) Find the absolute min/max of $f(x) = x-2\cos x$ on the interval $[-\pi,\pi]$.

(9) (a). Find the tangent line to $y = x^3$, when $x=1$.

(b) Find the area between the line from part (a), the graph of $y = x^3$ and the x axis, in the first quadrant.

(10) Find the volume of the solid resulting when the region in the first quadrant bounded by the graphs of

$y = 4x^2$ and $y = 16$ is revolved about the x-axis.

** <u>SET UP ONLY</u> - TWO WAYS**

(a) cylindrical shells

(b) disks/washers

(11) A balloon is rising vertically over a point A on the ground at a rate of 15 ft/sec. A point B on the ground is level with A and is 30 ft. from A. When the balloon is 40 ft. above A, at what rate is its distance from B changing?

(12) Find the equation of the line through (3,4) which cuts from the first quadrant a triangle of minimum area.

(13) Does the Mean Value Theorem apply to the given function? If so, find "c". If not, why not?

f(x) = $\sqrt{2x+1}$, [0,4]